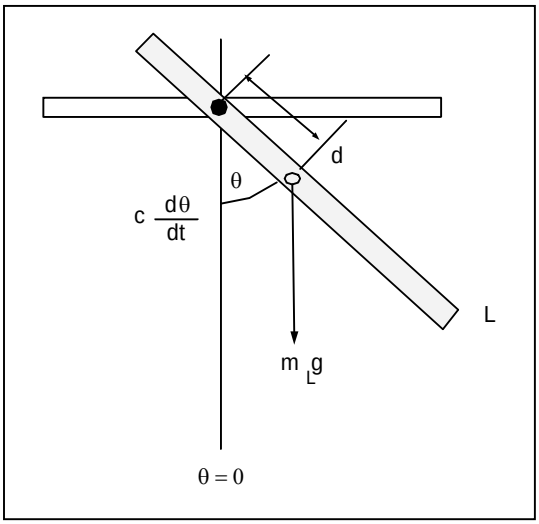
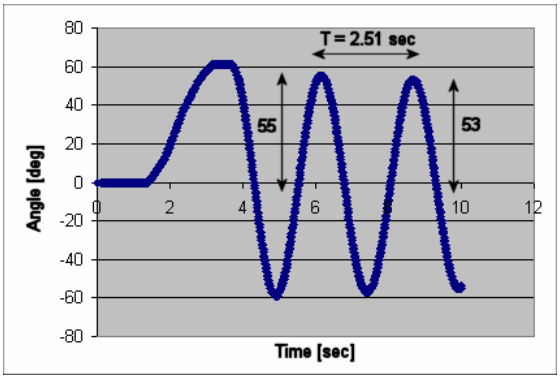
Homework 1 – Representations, Poles and Zeros

Below is a free-body diagram of the damped compound pendulum, dimensions and experimental data plotting the time response from free fall.



L	Bar length	0.495	m
d	Pivot to CG distance	0.023	m
m_L	Mass of pendulum	0.43	kg

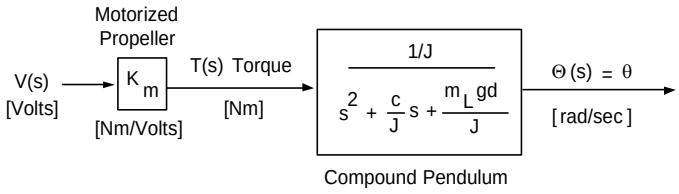


1. If c is the viscous damping coefficient, show that the equation of motion for the damped compound pendulum (sketch above) is given by the equation below (10 points)

$$\ddot{\theta} + \frac{c}{J}\dot{\theta} + \frac{m_L g d}{J}\theta = 0$$

2. Given the following block diagram and values, show that the open-loop transfer function is given by (5 points)

$$\frac{\Theta(s)}{V(s)} = \frac{1.89}{s^2 + 0.039s + 10.77} = G_{ol}(s)$$



J	$= 0.0090$	$kg \cdot m^2$
c	$= 0.00035$	Nms/rad
m_L	$= 0.43$	kg
d	$= 0.023$	m
g	$= 9.81$	m/s^2
K_m	$= 0.017$	Nm/V

3. Show that the time constant $T_c = 0.01475\,\text{sec}$ and settling time $t_s = 271.2\,\text{sec}$ for the damped compound pendulum given that $\zeta = 0.0059$ and natural frequency $\omega_n = 2.50\ rad/s$ (5 points)

4. Show that the complex poles $s_{1,2} = -2.4 \pm j2.4$ will yield $\omega_n = 3.39\ rad/s$ and phase angle $\theta = 0.785\ rad$ (5 points)