Attendance

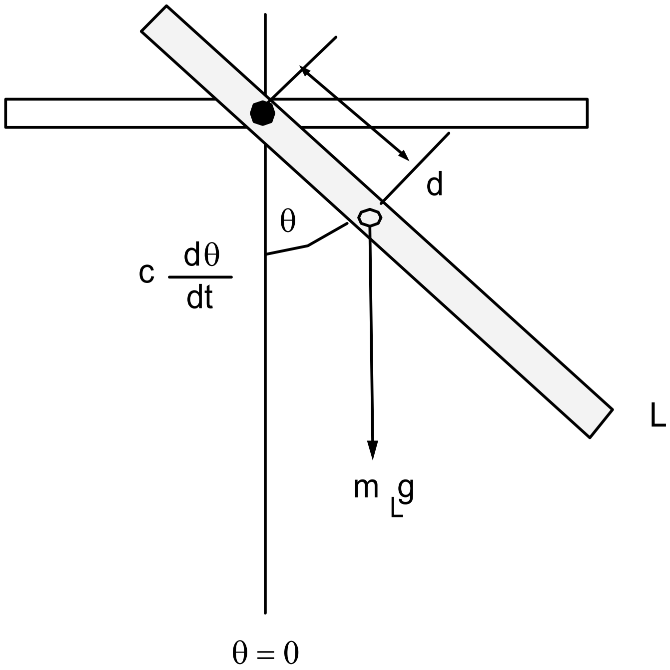
Survey Results

MEM 351 – Dynamic Systems Lab

Lecture 2: Transfer Functions, Poles, and Zeros

Recall

Damped Compound Pendulum Equations of Motion



$$\ddot{\theta} + \frac{c}{J}\dot{\theta} + \frac{m_L g d}{J}\theta = 0$$

Linearized 2nd order differential equation assumes **small** angles

L	Bar length [m]
d	Pivot to CG distance [m]
m_L	Mass of pendulum [kg]
J	Moment of Inertia $[kg \cdot m^2]$
c	Viscous damping coefficient $[\frac{Nms}{rad}]$

What do we expect ζ to be?

$$\ddot{\theta} + 2\varsigma\omega_n\dot{\theta} + \omega_n^2\theta = 0$$

Frequency Domain: Laplace Transforms

2nd order damped system

$$\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = 0$$

Laplace Transform

$$s^2\theta(s) + 2\zeta\omega_n s\theta(s) + \omega_n^2\theta(s) = 0$$

Yields

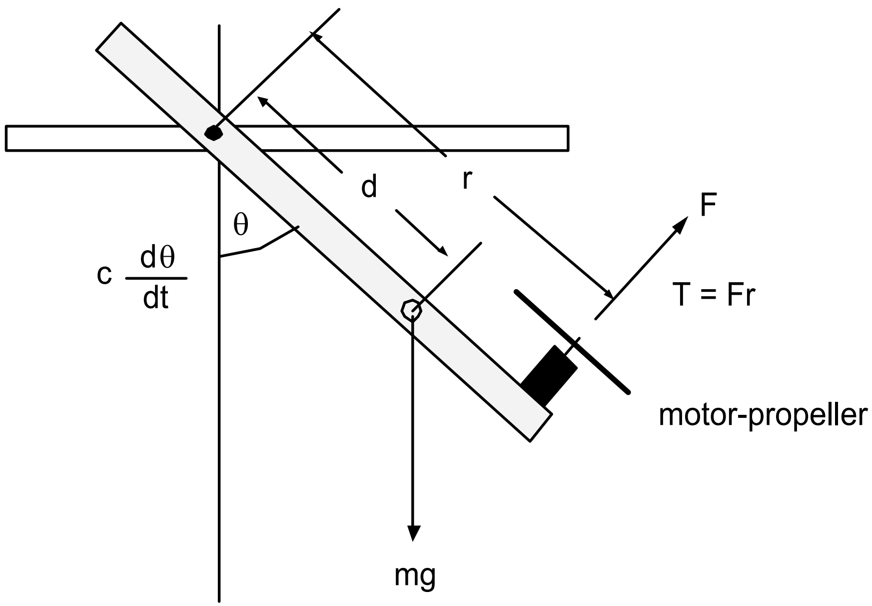
$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

Complex roots

$$-\zeta\omega_n \pm j\cdot\omega_n\sqrt{1-\zeta^2}$$

Nothing to control!!!

Torque: Something We Can Control?



- Voltage $V(s)$ applied to motor
- Propeller spins, creating lift force $F(s)$
- Lift on lever arm *r* creates torque $\mathrm{T}(s)$
- Pendulum angle defined by $\Theta(s)$

2nd order damped system

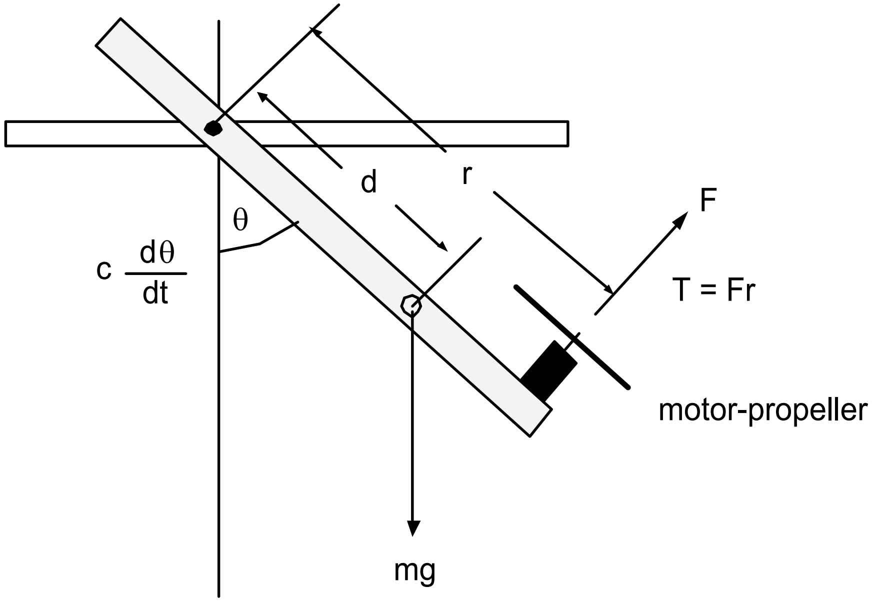
$$\ddot{\theta} + \frac{c}{J}\dot{\theta} + \frac{mgd}{J}\theta = T$$

Laplace Transform

$$\frac{\theta(s)}{T(s)} = \frac{1}{s^2 + \frac{c}{J}s + \frac{mgd}{J}}$$

Not actually controlling torque…

Voltage: Something We Can Control!



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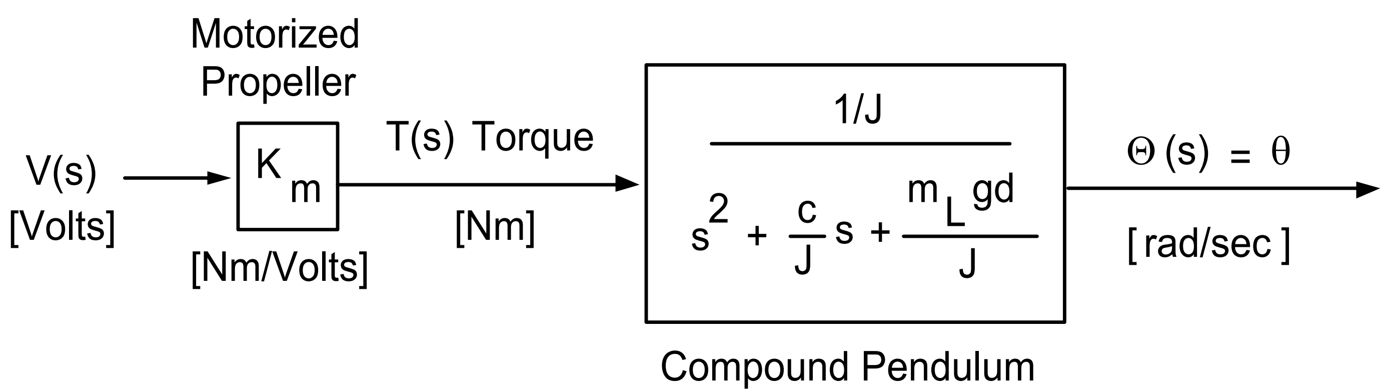
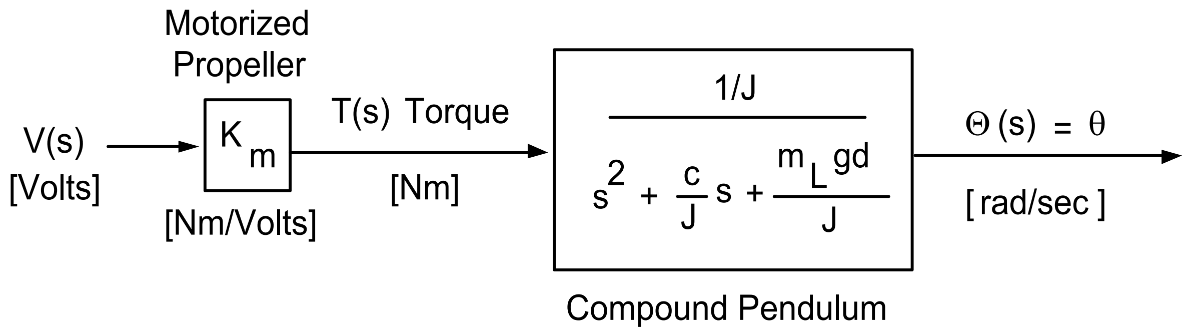


Calculating Constants



K_m Theoretically: can calculate lift force if have propeller pitch and radius dimensions, air density and motor angular velocity.

Experimentally: apply known voltage V and pendulum will eventually reach **steady-state**. Recall

$$J\ddot{\theta} + c\dot{\theta} + m_L g d \sin\theta = T$$

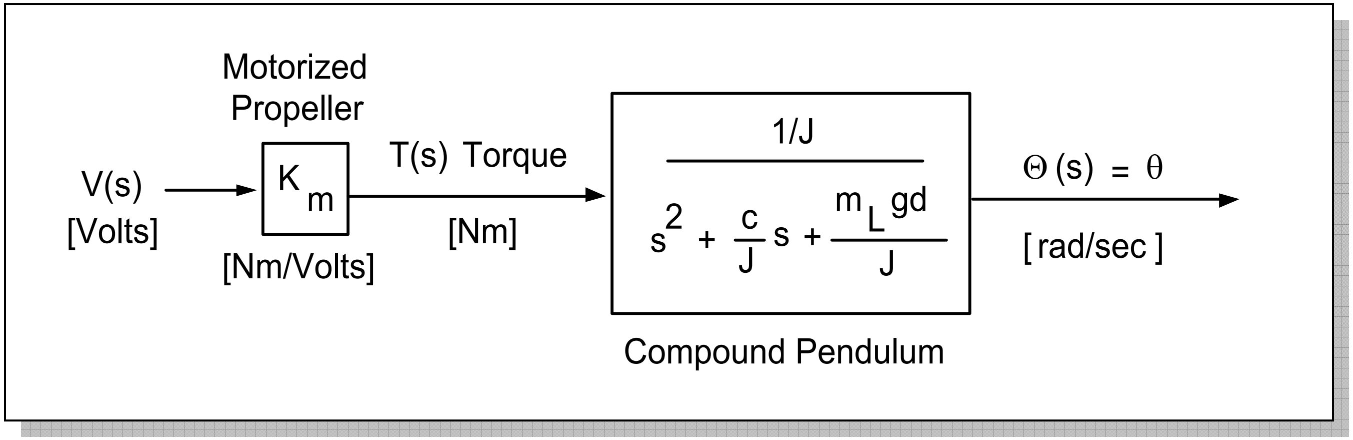
At steady-state angular acceleration and velocity are zero. The torque at this known voltage is calculated by:

$$T\big|_{ss} = m_L g d \sin\theta_{ss}$$

And hence

$$K_m = \frac{T\big|_{ss}}{V}$$

Open-Loop Transfer Function



OLTF: $$\frac{\Theta(s)}{V(s)} = \frac{K_m / J}{s^2 + \frac{c}{J}s + \frac{m_L g d}{J}} = G_{ol}(s)$$

Given

$K_m = 0.017$ Nm/V

$d = 0.023$ m

$J = 0.0090$ kgm^2

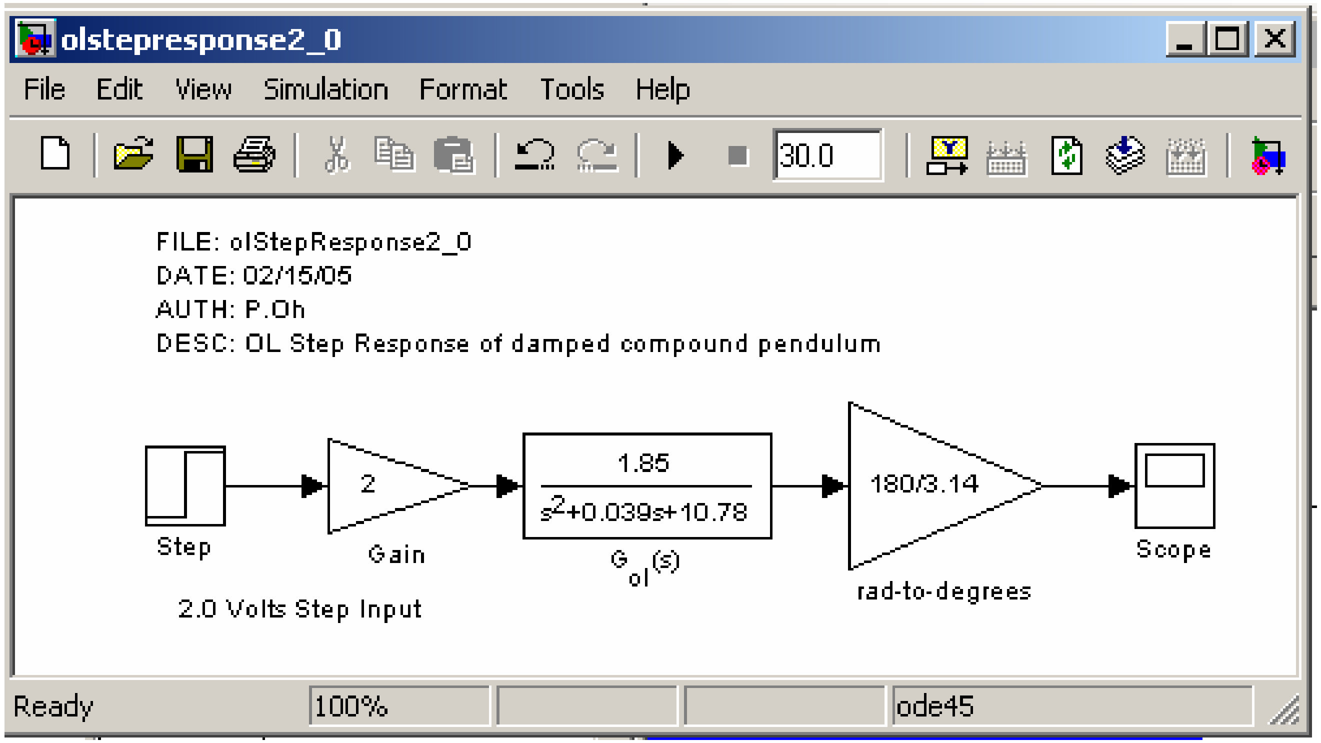
$m_L = 0.43$ kg

$c = 0.00035$ Nms/rad

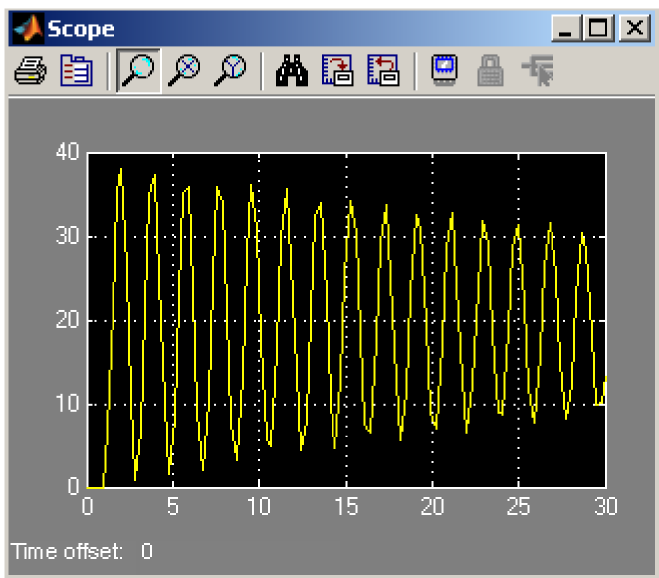
$$\frac{\Theta(s)}{V(s)} = \frac{1.89}{s^2 + 0.039s + 10.77} = G_{ol}(s) \qquad (3)$$

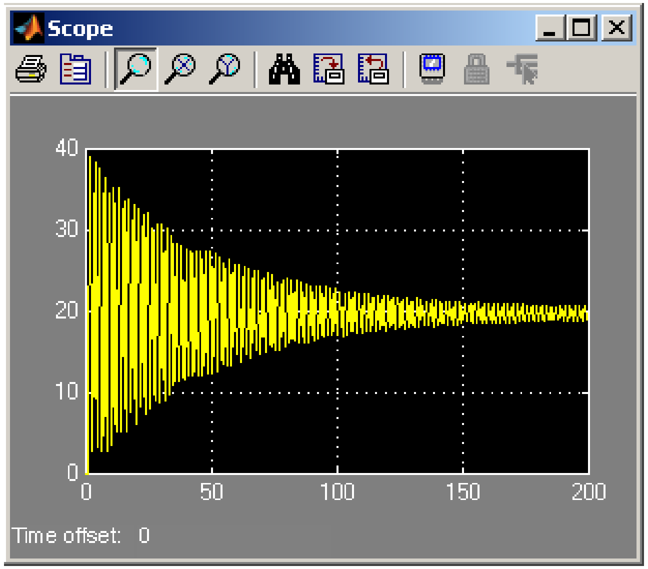
Laplace domain OL Transfer function

OLTF Simulations



Simulink





System Settling Time

Simulation reveals long settling time. This is consistent with the small damping ratio. Poles of the characteristic equation reveal the large oscillations. Recall from (1)

$$\frac{\Theta(s)}{V(s)} = \frac{1.89}{s^2 + 0.039s + 10.77} = G_{ol}(s)$$

Roots of the denominator (i.e. the poles) are:

$$s_1 = -0.0019 + j3.28$$
$$s_2 = -0.0019 - j3.28$$

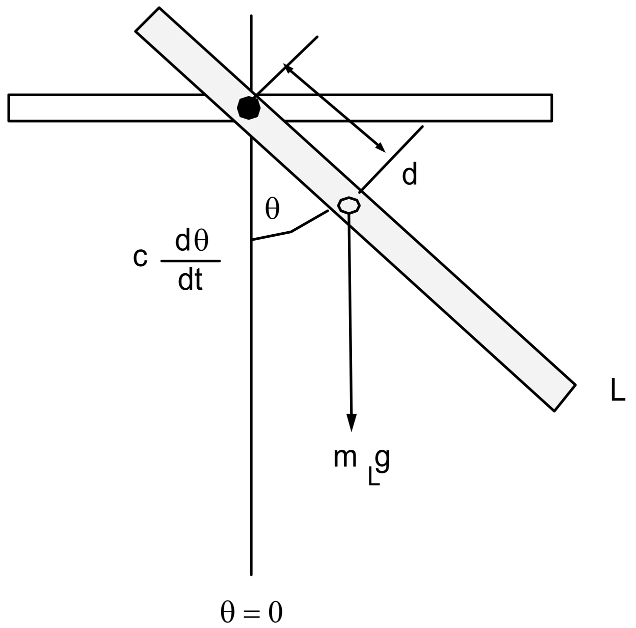
Small real root will yield long settling times

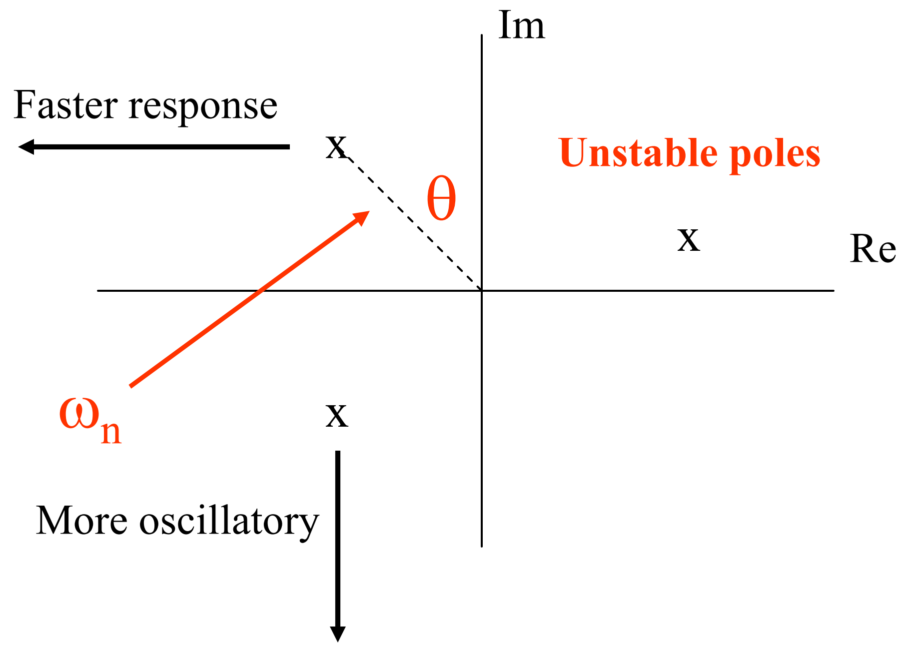
Can be shown:

Time constant $T_c = \zeta\omega_n$

2% settling time $t_s = 4 / T_c$

Effect of System Poles





Control Designer's Goal:

Create compensators that yield desired damping and rise time.

In other words, place poles where one wants them

Side Note:

$$s_{1,2} = -0.0019 \pm j3.28$$

Find ω_n and phase angle θ

Homework #1

DUE TUESDAY 01/22/07

Tedious Math: Time domain differential equation

2nd order damped system $\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = 0$

Yields complex roots $-\zeta\omega_n \pm j\cdot\omega_n\sqrt{1-\zeta^2}$

Time domain solution

$$\theta_c(t) = e^{-\zeta\omega_n t}\left\{A_1\cos\left(\omega_n t\sqrt{1-\zeta^2}\right) + A_2\sin\left(\omega_n t\sqrt{1-\zeta^2}\right)\right\} \quad (1)$$

Small real root will yield long settling times

Can be shown:

Time constant $T_c = \zeta\omega_n$

2% settling time $t_s = 4/T_c$ (2)