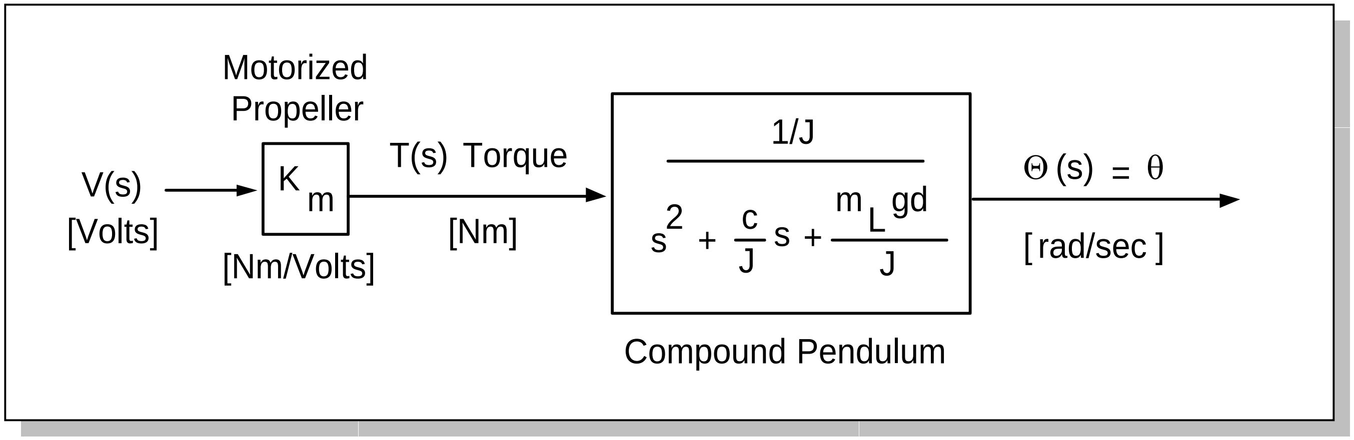
Today’s Outline

1. Attendance
2. Homework 1
3. Lecture
4. Form Lab Groups
5. Lab 2

MEM 351 – Dynamic Systems Lab

State Space Realizations

Recall Last Week: Open-Loop Transfer Function



V(s) [Volts]

Motorized Propeller

K_m

[Nm/Volts]

T(s) Torque [Nm]

$$\frac{1/J}{s^2 + \frac{c}{J}s + \frac{m_L g d}{J}}$$

Compound Pendulum

$\Theta(s) = \theta$ [rad/sec]

OLTF:

$$\frac{\Theta(s)}{V(s)} = \frac{K_m / J}{s^2 + \frac{c}{J}s + \frac{m_L g d}{J}} = G_{ol}(s)$$

Given

$K_m = 0.017$ Nm/V

$d = 0.023$ m

$J = 0.0090$ kgm^2

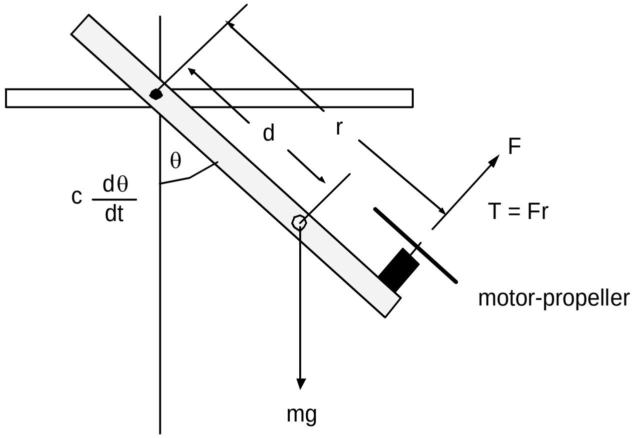
$m_L = 0.43$ kg

$c = 0.00035$ Nms/rad

$$\boxed{\frac{\Theta(s)}{V(s)} = \frac{1.89}{s^2 + 0.039s + 10.77} = G_{ol}(s)} \quad (1)$$

Laplace domain OL Transfer function

State Space Formulation (I)



Equation of Motion:

$$J\ddot{\theta} + c\dot{\theta} + m_L g d\theta = T$$

Substitute (T = K_mV):

$$\ddot{\theta} + \frac{c}{J}\dot{\theta} + \frac{m_L g d}{J}\theta = \frac{K_m}{J}V \qquad (2)$$

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

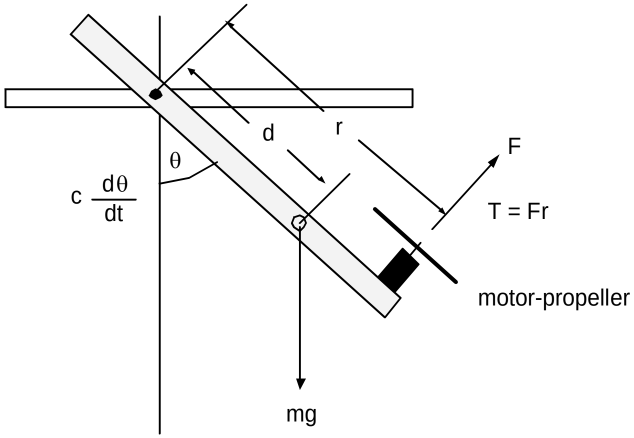
x: state vector

$\dot{x}$: derivative of state vector

y: output vector

u: input or control vector

State Space Formulation (II)



$$\ddot{\theta}+\frac{c}{J}\dot{\theta}+\frac{m_L g d}{J}\theta=\frac{K_m}{J}V \qquad (2)$$

Suppose we define two state variables (why two?):

$$x_1=\theta \quad \text{and} \quad x_2=\dot{\theta}$$

We can re-write (2) as

$$\dot{x}_1=x_2$$

$$\dot{x}_2=-\frac{c}{J}x_2-\frac{m_L g d}{J}x_1+\frac{K_m}{J}V \qquad (3)$$

State Space Formulation (III)

State space form given by matrices:

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + du\end{aligned} \qquad (4)$$

Re-expressing

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{c}{J}x_2 - \frac{m_L gd}{J}x_1 + \frac{K_m}{J}V$$

Gives

$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1 \\ -\frac{m_L gd}{J} & -\frac{c}{J}\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}0 \\ K_m / J\end{bmatrix}u \qquad (5)$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + 0 = x_1 = \theta$$

Finding the System Poles from State Space

The state space representation (5) and transfer function (1) are just the same representation of the real-world system. As such, they should all have the same characteristic equation (i.e. same poles)

The characteristic equation from state space (4) is defined as:

$$\alpha(s) = \det(sI - A) = 0 \qquad (6)$$

$$\alpha(s) = \det\left[\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -\dfrac{m_L gd}{J} & -\dfrac{c}{J} \end{pmatrix}\right]$$

$$= \det\left[\begin{pmatrix} s & -1 \\ 10.77 & s+0.039 \end{pmatrix}\right] = s^2 + 0.039s + 10.77 \qquad (7)$$

Which is the same as the denominator given in (1)

$$\frac{\Theta(s)}{V(s)} = \frac{1.89}{s^2 + 0.039s + 10.77} = G_{ol}(s)$$

How to Sample a Signal

Sampling

Defined:

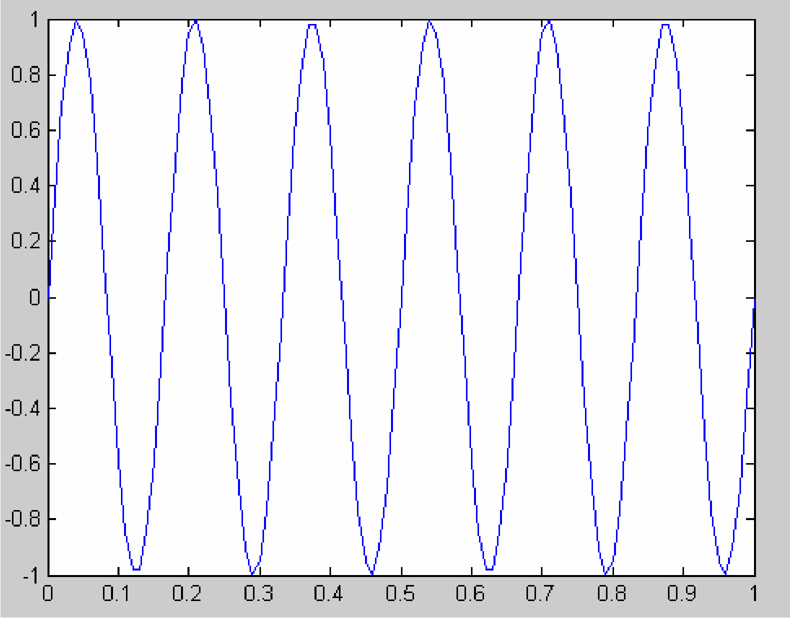
- The process of converting a continuous time signal into a discrete time signal
- The frequency the incoming signal is sampled at is very important (aliasing)!!

Example:

- Open up MATLAB
- Click on File => New and enter in the following code

```
t=0:0.01:1;
y=sin(6*2*pi*t);
plot(t,y);
```

- Hit F5 to run it



What is the frequency of this sine wave?

What is the sample frequency?

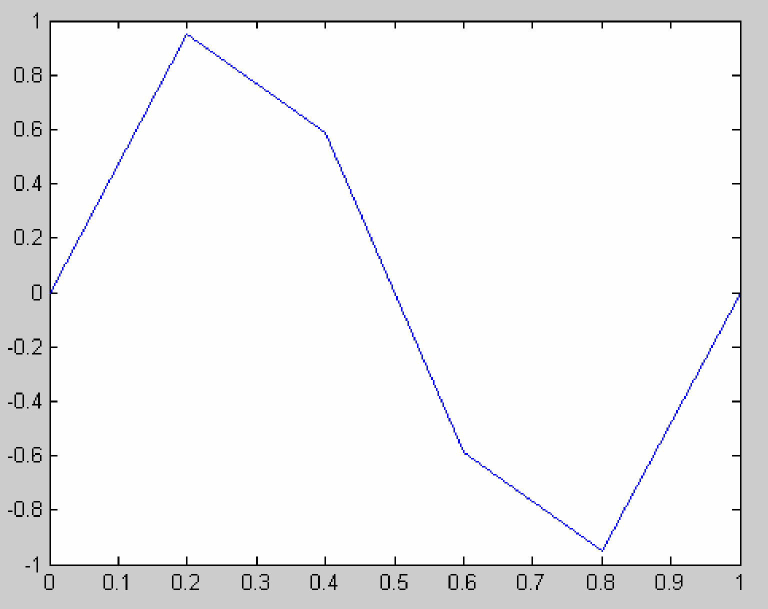
Sampling

What will happen if we sample the same 6 Hz sine wave at 5 Hz?

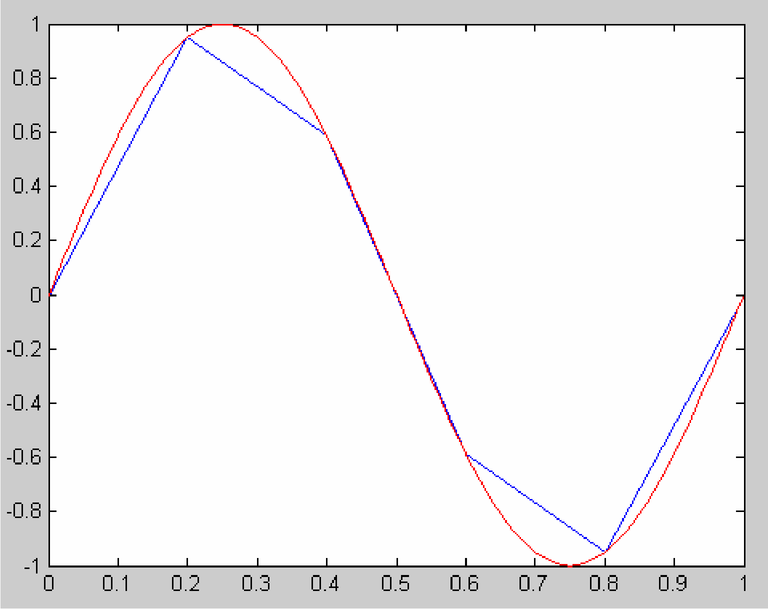
- Modify the code as follows

```
t=0:0.2:1;
y=sin(6*2*pi*t);
plot(t,y);
```

- Hit F5 to run it



It appears to be a 1 Hz sine wave => aliasing!



Shannon’s Theorem

The sampling frequency must be at least twice that of the highest frequency

- In our case, the highest frequency is 6 Hz
- So our sampling frequency must be at least 12 Hz
- Modify the code to incorporate the variable fs = 12 (sampling frequency)

```
fs = 12;
t=0:(1/fs):1;
y=sin(6*2*pi*t);
plot(t,y);
```

- This is still not good enough
- In reality, the sampling rate should be 5-20 times the highest frequency
- Play with the sampling frequency until you get a nice smooth graph

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