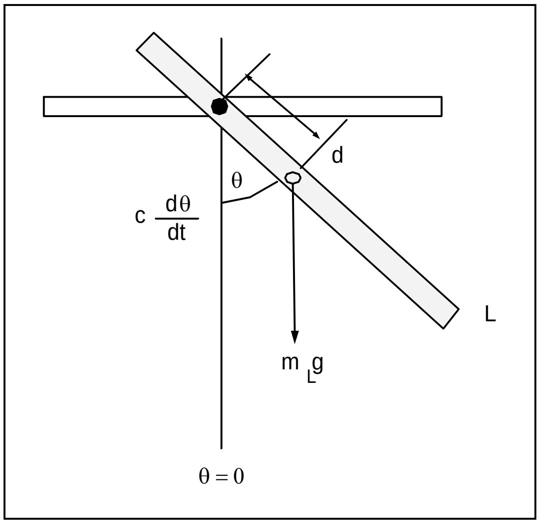
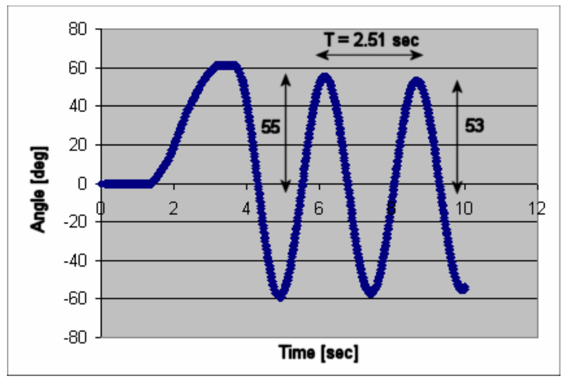
Homework: Modeling and System Identification

Below is a free-body diagram of the damped compound pendulum, dimensions and experimental data plotting the time response from free fall.



J	Moment of Inertia	0.0090	kgm^2
L	Bar length	0.495	m
d	Pivot to CG distance	0.023	m
m_L	Mass of pendulum	0.43	kg



1. Use the experimental data above to show that $\zeta = 0.0059$ and $\omega_n = 2.50$ rad/sec. (5 points)

2. The equation for the experimental data is given by $\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = 0$. Compare this to equation of motion $\ddot{\theta} + \frac{c}{J}\dot{\theta} + \frac{m_L gd}{J}\theta = 0$ to show that $c = 0.00035\ \frac{N \cdot m \cdot s}{rad}$ and $\omega_n = 3.28\ \frac{rad}{s}$ (5 points)

3. Include an Excel plot of the data you acquired in Lab (where you used the encoder to capture angle data). Label consecutive peaks and period. Show calculations for ζ and ω_n (15 points)

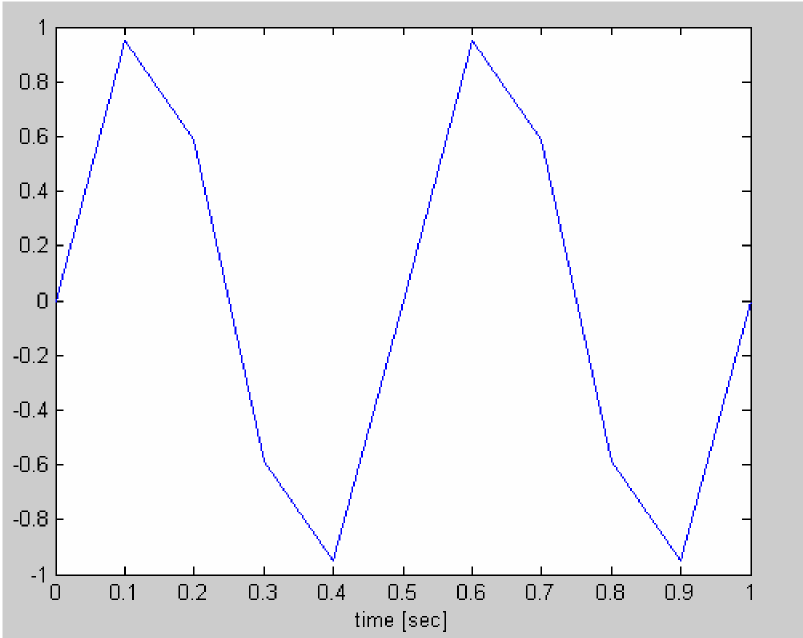
4. Given the following differential equation

$$\dddot{\theta} + 10\ddot{\theta} + \theta = \frac{K_m}{J}V$$

Determine the state space representation given that V and θ are the input and output of the system, respectively. That is, put the above equation in the following form (7.5 points)

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

5. If the equation of a sine wave is given as y = sin(24πt), what is the frequency of the signal? Plotting the above equation in MATLAB yields the following graph:



Is the frequency of the signal in the above figure equivalent to your answer in the first part of question 5? If not, please explain this phenomenon. Also, write MATLAB code to *correctly* plot the above equation and include your code and graph. (7.5 points)