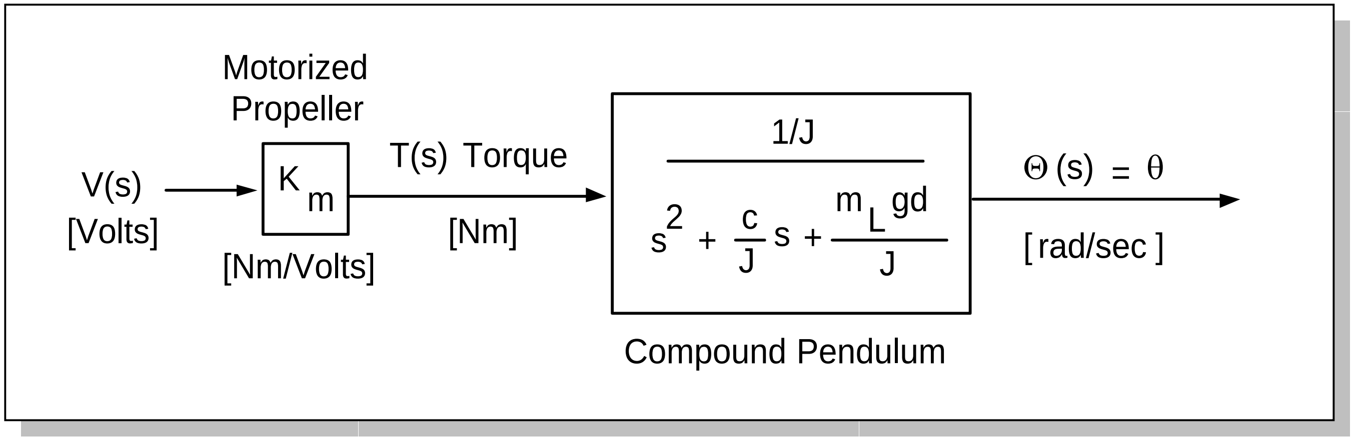
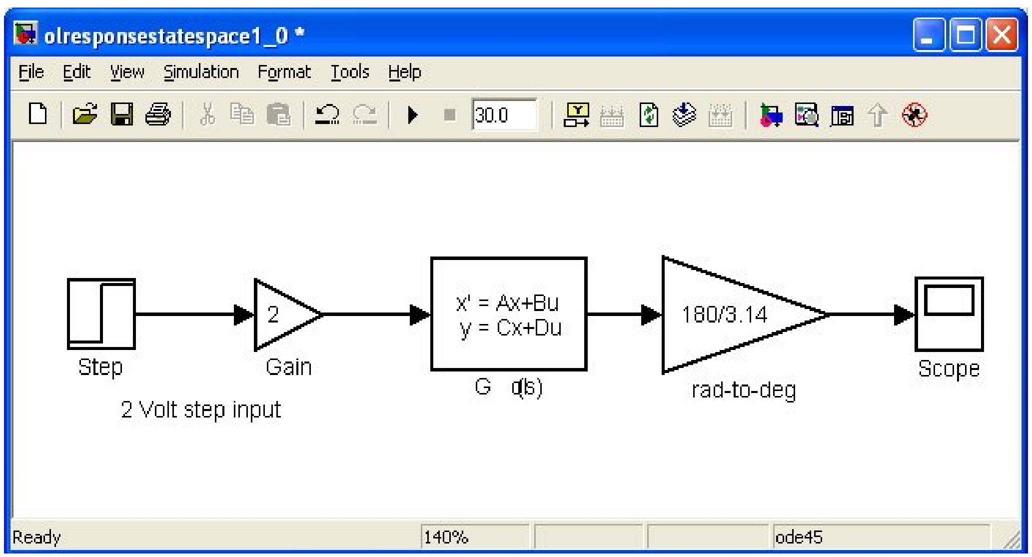
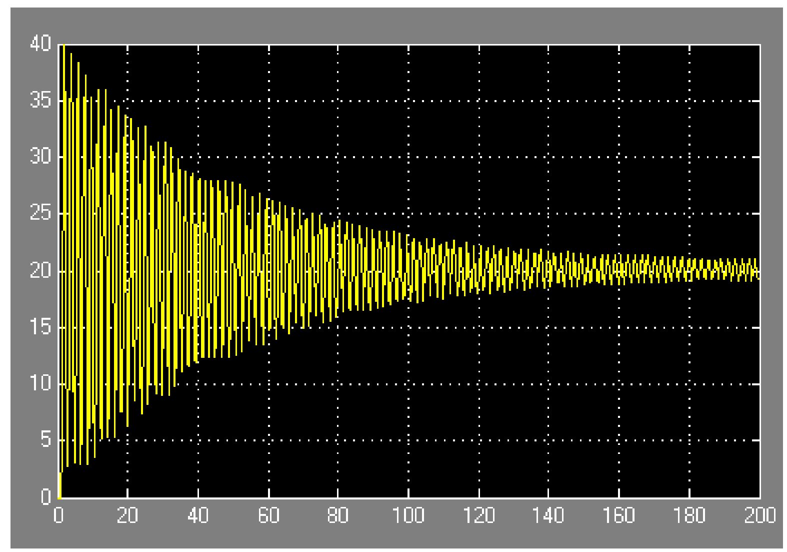
MEM 351 – Dynamic Systems Lab

Control Design 1: Pole-placement

Recall: Open-Loop Transfer Function





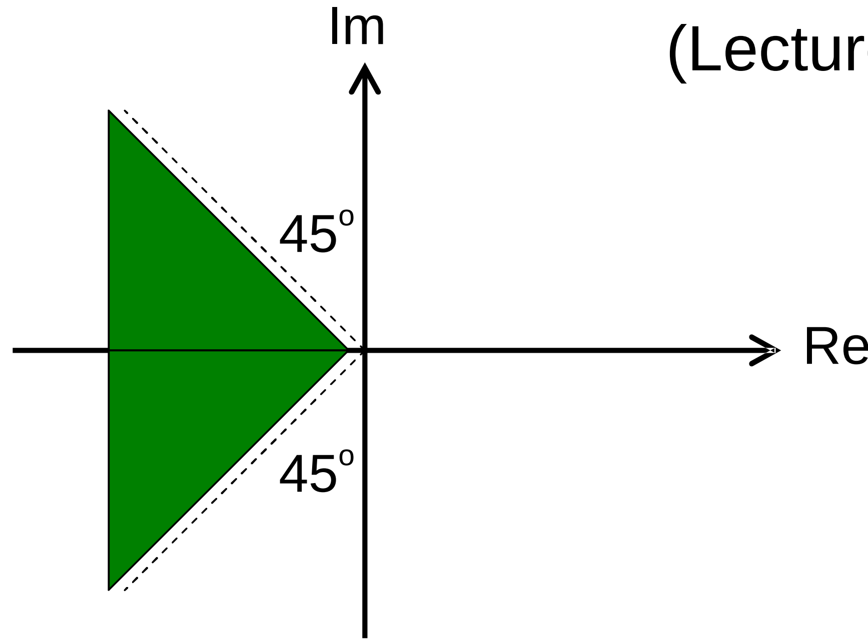


Need for control!

Recall: Pole Locations ⇔ System Stability

(Lecture 2)

If the system is controllable, then poles of the closed-loop system may be placed at any desired location



If our desired pole locations are represented by s_1, s_2, …s_n, then our desired characteristic equation, α_d , is

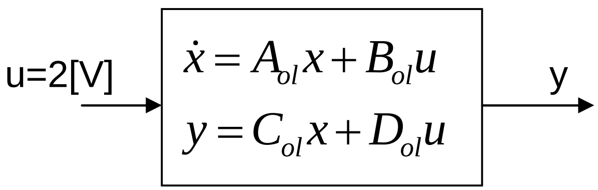
$$\alpha_d = (s - s_1)(s - s_2)\cdots(s - s_n) = 0 \qquad (1)$$

$$\alpha_d = s^2 + \alpha_1 s + \alpha_0 = 0 \qquad (2)$$

How can we force this to be our system's characteristic equation?

Pole Placement: Characteristic Equation

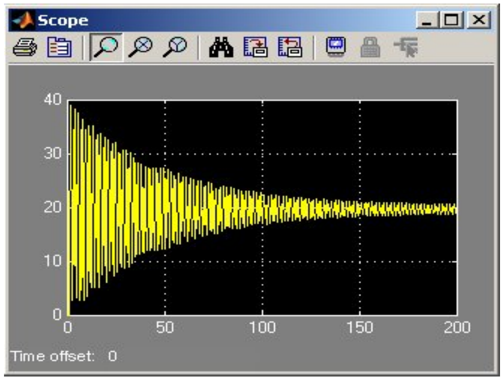
Open-Loop System



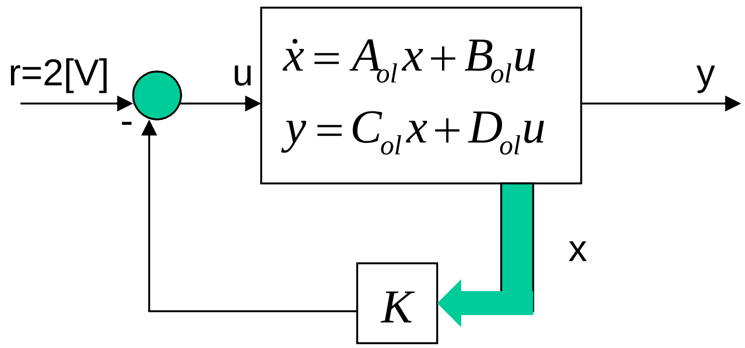
$$\dot{x} = A_{ol}x + B_{ol}u$$

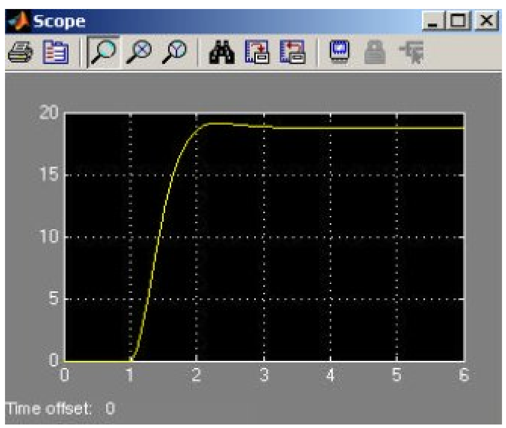
$$y = C_{ol}x + D_{ol}u$$

$$\alpha(s) = \det(sI - A)$$



Feedback System using Pole Placement





What is the characteristic equation of this?

Pole Placement: Equations

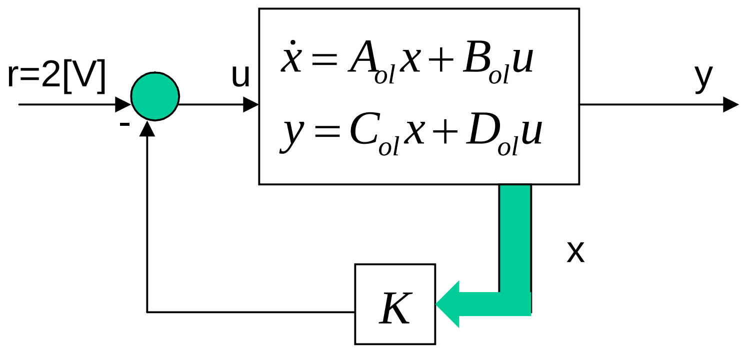
In standard state space form, the characteristic equation is

$$\dot{x} = A_{ol}x + B_{ol}u \quad \Rightarrow \quad \alpha(s) = \det(sI - A_{ol}) = 0$$

Our input is now

$$u = r - Kx$$

$$\dot{x} = A_{ol}x + B_{ol}(r - Kx)$$



Simplifying, we have

$$\dot{x} = (A_{ol} - B_{ol}K)x + B_{ol}r$$

The new characteristic equation is

$$\alpha_s(s) = \det(sI - (A_{ol} - B_{ol}K)) = 0$$

$$\alpha_s = s^2 + a_1 s + a_0 = 0 \qquad (3)$$

Pole Placement: Equations

One then matches the coefficients in (2) with those of (3) to yield values for gains

$$\left.\begin{aligned} \alpha_d &= s^2 + \alpha_1 s + \alpha_0 = 0 \\ \alpha_s &= s^2 + a_1(k_2)s + a_0(k_1) = 0 \end{aligned}\right\} \quad K = \left[k_1 \ldots k_n\right]$$

Pole Placement Control

An Example

Pendulum: Desired Poles

Suppose one wants a settling time of $t_s = 1.67 \text{ sec}$
and a damping ratio $\zeta = 0.707$

$$t_s = 4/\zeta\omega_n$$

$$s_{re} = -\zeta\omega_n$$

$$s_{im} = j\omega_n\sqrt{1-\zeta^2}$$

This results in poles for the damped compound pendulum

$$s_{1,2} = s_{re} \pm s_{im} = -2.4 \pm j2.4$$

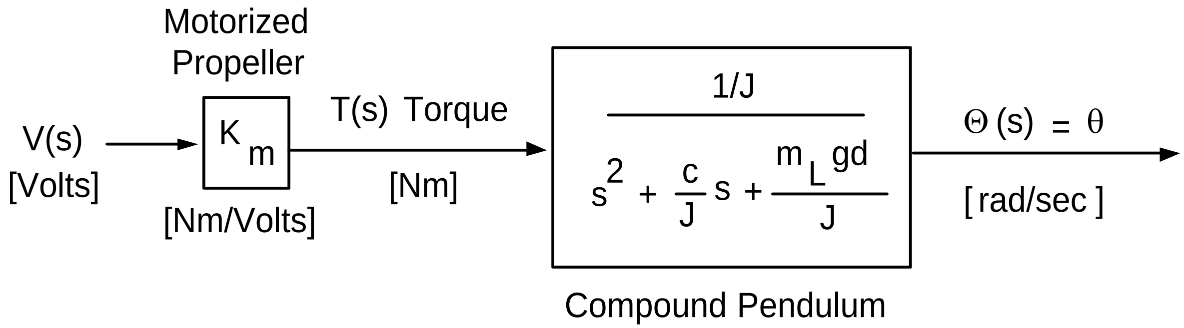
Substituting the desired poles in (2) yields

$$\alpha_d = (s + 2.4 + j2.4)(s + 2.4 - j2.4)$$

$$\alpha_d = s^2 + 4.8s + 11.52 \qquad (4)$$

Desired characteristic equation = System characteristic equation

Pendulum: State Space Realization (Lecture 3)



Given $\ddot{\theta} + \frac{c}{J}\dot{\theta} + \frac{m_L g d}{J}\theta = \frac{T}{J}$ then $\ddot{\theta} + \frac{c}{J}\dot{\theta} + \frac{m_L g d}{J}\theta = \frac{K_m}{J}V$

Putting this in state space form yields (open loop)

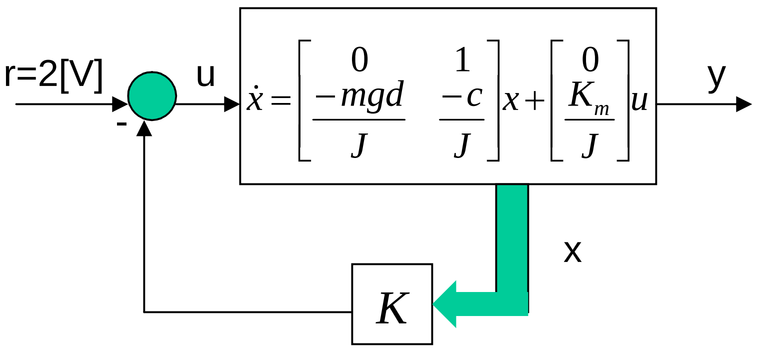
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{m_L g d}{J} & -\frac{c}{J} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ K_m / J \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0 = x_1 = \theta$$

Pendulum: Characteristic Equation

The characteristic equation of the closed loop system is calculated from

$$\alpha_s(s) = \det(sI - (A_{ol} - B_{ol}K)) = 0$$



which becomes

$$\det\left\{\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \left(\begin{bmatrix} 0 & 1 \\ -\frac{m_L gd}{J} & -\frac{c}{J} \end{bmatrix} - \begin{bmatrix} 0 \\ K_m / J \end{bmatrix}\begin{bmatrix} k_1 & k_2 \end{bmatrix}\right)\right\}$$

$$\det\begin{bmatrix} s & -1 \\ \frac{m_L gd + K_m k_1}{J} & s + \frac{c + K_m k_2}{J} \end{bmatrix} = s^2 + \underbrace{\left(\frac{c + K_m k_2}{J}\right)}_{a_1(k_2)} s + \underbrace{\frac{m_L gd + K_m k_1}{J}}_{a_0(k_1)} = 0 \qquad (5)$$

Pole Placement: Gains

Setting the coefficients in equations (4) and (5) equal

$$\alpha_d = s^2 + 4.8s + 11.52$$

$$\alpha_s = s^2 + \left(\frac{c + K_m k_2}{J}\right)s + \frac{m_L gd + K_m k_1}{J}$$

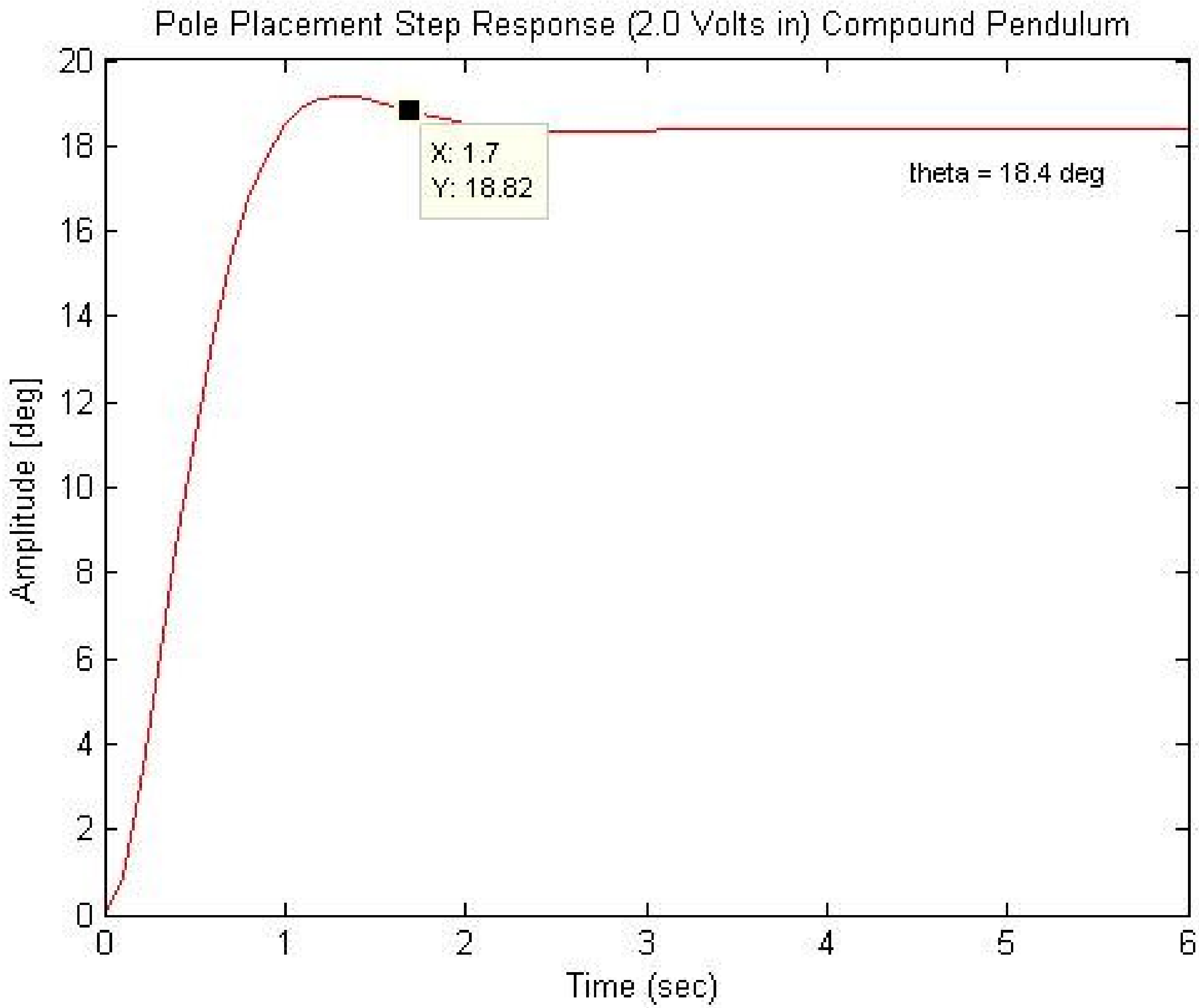
yields

$$\frac{c + K_m k_2}{J} = 4.8$$

$$\frac{m_L gd + K_m k_1}{J} = 11.52$$

$$k_2 = \frac{4.8J - c}{K_m}$$

$$k_1 = \frac{11.52J - m_L gd}{K_m}$$



Slight overshoot and reduced settling time – inline with desired response

MEM 351 – Dynamic Systems Lab

Control Design 2: PID

PID Controller

Background

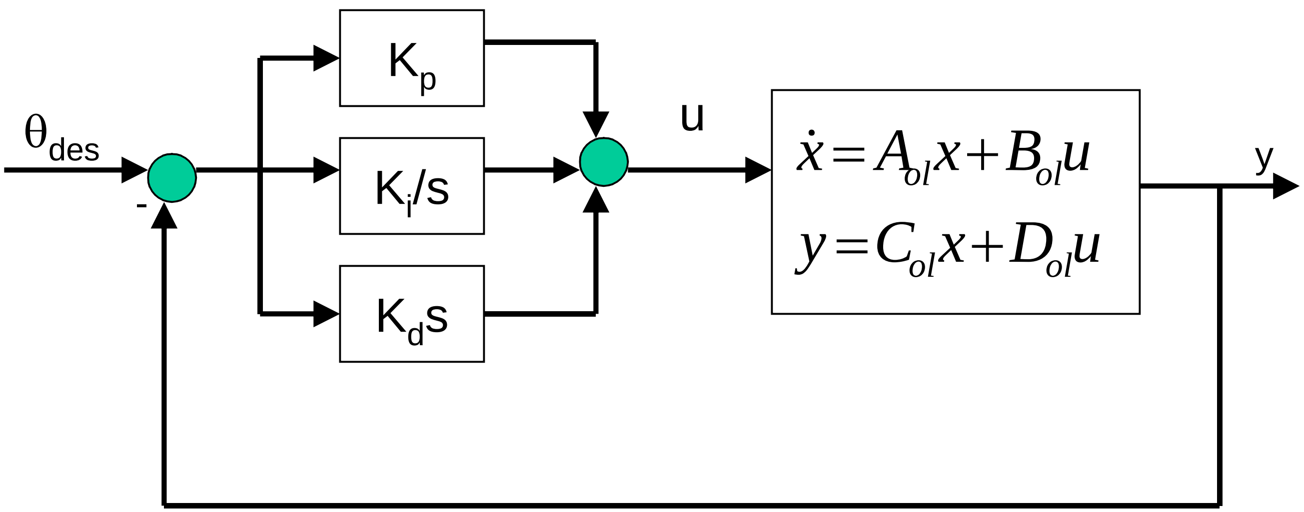
- More than 50% of the industrial controllers in use today utilize PID or modified PID control schemes
- Can be utilized as a control technique without knowledge of the plant's mathematical model

Components

- Proportional: used to handle the present
- Integral: handles the past by integrating the error over time
- Derivative: handles the future by taking the first derivative over time

PID Controller

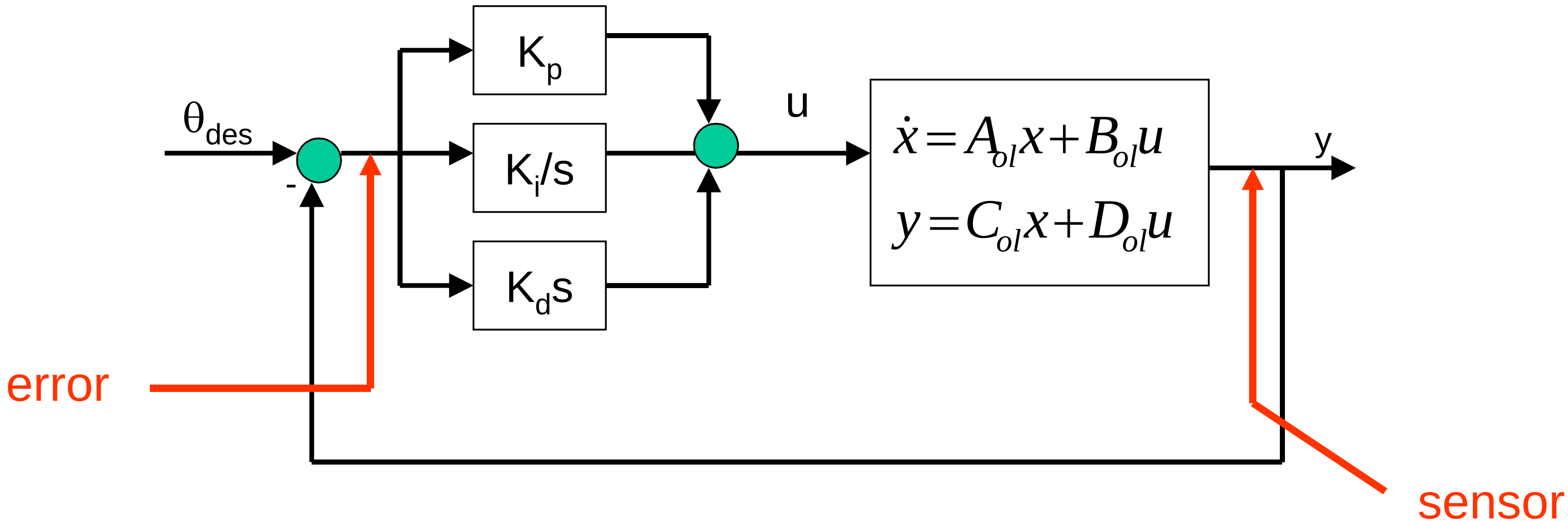
A PID controller in block diagram form



has the following transfer function

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s = \frac{K_p s + K_i + K_d s^2}{s}$$

How it Works?



Steps

1. Compute the error (desired – actual)
2. Multiply the error by proportional constant K_p
3. Integrate the error and multiply by K_i
4. Take the derivative of the error and multiply by K_d
5. Sum together to form input u

Tuning Parameters

Parameters	Rise Time	Overshoot	Settling Time	SS Error
P	Decrease	Increase	Small Change	Decrease
I	Decrease	Increase	Increase	Eliminate
D	Small Change	Decrease	Decrease	Small Change